Structural Causal Models – SCM

Sergio Focardi, PhD

We espouse a concept of causality related to the concept of *intervention*. Superficially intervention seems to be similar to setting initial conditions in a system of differential equations. However, there is an important conceptual difference. Differential equations are logical structures and setting initial conditions is a purely logical operation that adapts the equations to a given system. Intervention means that the process is changed by some intervening action. Interventions cannot be reversed.

Correlations imply causation in the following sense. Suppose X and Y are two correlated random variables. There are three possibilities: 1) X has a causal effect on Y or 2) Y has a causal effect on X or 3) there is a common cause for both variables. The third condition is called the Reichenbach Common Cause Principle (RCCP) because it was formulated by Hans Reichenbach in 1956. RCCP is widely used in practice, but it fails in a number of circumstances related to quantum mechanics.

An important property of causal networks is the Causal Markov Condition a.k.a. Causal Markov Assumption. The causal Markov condition applies to a distribution *p* and to a DAG G:

A distribution *p* is said to satisfy the Markov Condition with respect to a DAG G if each variable *X is conditionally independent of all its non-descendants given its parents PA(X)*.

Models

Let's describe the SCM (Structural Causal Model). Assume V={X1,X2,...,Xn} be a set of endogenous variables, and U={U1,U2,...,Un} be corresponding exogenous variables. Assume that each endogenous variable Xi be a function of its parents PA(Xi) and of Ui:

$$X_i := f_i(PA(X_i), U_i).$$

This equation is a causal assignment in the sense that the Xi variable is supposed to assume the value fi(PA(Xi),Ui). However, this relationship is not a standard invertible equality between the left and the right part of the equation. Suppose that the endogenous variables are independent random variables. The set *M* of these functional relationships is called a SCM model.

Any SCM *M* induces a causal graph. The SCM *M* induces the causal graph G where each X is a vertex (node) and there is a directed edge from each node in PAi to Xi for each i. Given a SCM *M*, acyclicity is the property that the induced graph G is a DAG, that is, it does not contain cycles. The independence of the Ui so that p(U1,...,Un)=p(U1)x...xp(Un) is called causal sufficiency

Let's remark that a causal model is an abstract logical structure that links variables. A causal model does not attempt to explain why a variable has a causal effect on another variable. This fact applies to all physical models. Scientific explanation in general means that observations can be inferred from a theory through a purely logical process. This notion is widely accepted by physicists. A newly discovered phenomenon is explained if it can be logically deducted by a theory. The philosopher Carl Hempel formalized this notion and put forward what he called the Deductive-Nomological, DN, theory of scientific explanation.

Causal models are characterized by a mathematical form where variables appear only once on the left of equations. This is the most common form of SCMs. The mathematical structure of SCMs allows to represent them as DAGs. We will now describe some additional properties of SCMs and the assumptions needed to infer SCMs from probability distributions.

Interventions and counterfactuals

An intervention consists in replacing a structural equation with a given value. Given an SCM *M*, an intervention, represented by the symbol do(Xi:=xi), is modelled by replacing the i-th structural equation Xi:=fi(PA(Xi),Ui) with the assignment Xi:=xi. An intervention induces a new distribution and a new graph G'.

Counterfactual reasoning are statements about what would have been observed after some interventions. Counterfactuals are computed in three steps: 1) compute the noise distributions given the actual observations, 2) implement some interventions, and 3) compute the quantities of interest under the same conditions of the original observations.

Independent Causal Mechanism

Causal modelling assumes that the causal mechanisms are independent of each other. Citing an often-used example, altitude has a causal effect on temperature, that is, temperature changes in function of the altitude. We assume that this mechanism does not change in function of position. For example, we assume that the causal physical mechanism responsible for the change of temperature is the same in Austria and Switzerland.